SANDEEP SRIVASTAVA'S

ALL THAT MATTERS IN MATH

Chapter K

The Family Edition

Volume I
KG-VIII READING BOOK

SHARPENING HABITS OF THINKING

ALL THAT MATTERS IN
MATH

A to Z of Mathematics

The Family Edition

Volume I

KG-VIII Reading Book

SANDEEP SRIVASTAVA

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Volume - I

KG-VIII READING BOOK

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Dedicated to Reeta and Krishan Tikoo, for enabling conditions for this book to be authored.

“The word ‘mathematics’ comes from the root word manthanein, meaning ‘learn’; it implies ‘that which is learnable’.

But what have we made of math education – almost unlearnable ‘school math’?”

“The structure and role of family withstood the age of TV and its onslaught of entertainment, but they fell apart like a house of cards during the Internet age!

The evidence of a crumbling family edifice is reflected in every crisis that we are living with – old-age issues, mental health, emotional health, family discord, increased bullying of children, increasing scope of school in education, etc.

There is just one elixir to reinvigorate every family across the globe – making legacy educational disparities among families inconsequential to imparting math education to the next generation.”

“Children’s success in math is the proverbial ‘tasting of blood’ for further academic excellence; success in math in school releases a lot of time, attention, and effort spent on chasing math grades.” *It’s unnatural for human children to be struggling in math.*

“In case the severity of math education deficit isn’t obvious, consider what would happen if schools, not parents, were to ‘teach kids how to walk’.” *Indeed, math can’t be taught in classrooms.*

“The way math is taught in schools is of inexcusably poor quality, the evidence – being ‘poor in math’ is one sign of creative geniuses, the only socially acceptable ‘failure’, globally.

Teaching math the wrong way starts from pre-school. Had you been taught well, you would have fallen in love with math, and your life and career would have benefited from a happy association with math.”

Contents

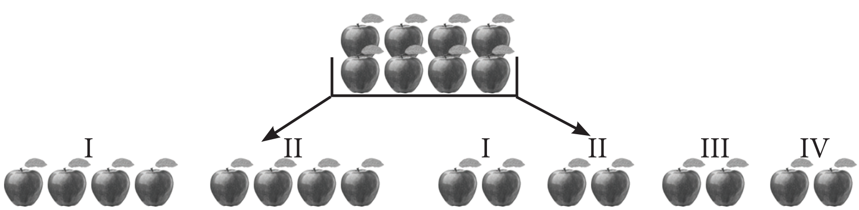
K for Knife

Making groups/packages is routine, exploring division

Making smaller groups out of bigger groups such as 4 bags of 2 apples each from a sack of 8 apples, or making bigger/more groups out of smaller quantities (e.g., making 16 half apples out of 8 apples) are commonplace activities for us. Welcome to division, the concept and practice of knifing things into smaller, equal parts or groups.

Revisiting unit of counting

Unit of counting is everything in quantification – the magnitude of things changes with the unit; a quantity of 12 eggs is just 1 dozen eggs. Changing the unit of counting/measurement is like changing the size of the group formed in a given quantity of things. Here is how 8 apples can be grouped in 2 ways:



Unit of counting is 4 apples; these are 2 groups of 4 apples

Unit of counting is 2 apples; these are 4 groups of 2 apples

Counting and measurement can also be seen as a way of reorganising or regrouping any given quantity of things.

The science of regrouping things

As we have a dedicated operation that helps us make groups out of a given quantity of things (for example, make 5000 1-litre bottles of shampoo from 5000 litres of shampoo in a bulk container), counting and measurement may not be used for creating groups. We call this 'regrouping' a given set of things because we change the way things are placed/held/seen. Another way of regrouping given set of things is to make many parts of each such set (for example, slicing a cake).

The operation, called 'division', is a complete 'science' of regrouping things.

What is division?

Division is a specific way of regrouping things – all the groups to be created are of the same size. Not every regrouping of things may be called division of things. For example, 10 balls can be regrouped in multiple ways as follows:

Figure 1: 1 group of 10 balls

Figure 2: 3 groups of 4, 4, and 2 balls

Figure 3: 4 groups of 3, 3, 3, and 1 ball

Figure 4: 2 groups of 5, 5 balls

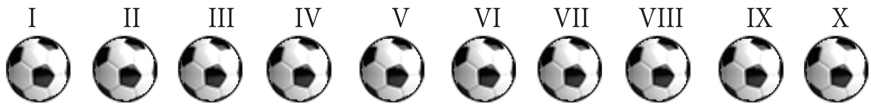


Figure 5: 10 groups of 1 ball

The figures 1, 4, and 5 show the division of 10 balls into equal groups, and figures 2 and 3 show just the grouping of 10 balls (not division).

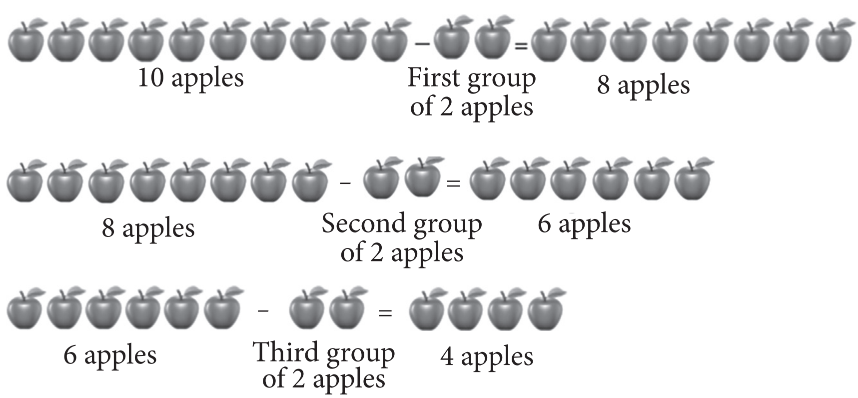
The division operation

There are four important aspects when we consider at regrouping a given quantity of things:

1. The exact quantity of a given thing. This quantity is called 'dividend'.
2. The exact quantity of each group the given thing must be separated into. This quantity is called 'divisor'.
3. The exact quantity of the same-sized groups. This quantity is called 'quotient'.
4. The exact quantity of given things which couldn't be grouped, if any at all. This quantity is called 'remainder'; Obviously, the remainder will be a quantity that is less than the quantity of divisor.

Visualising division operation

Let's visualise how we may divide 10 apples into groups of 2 apples.



Thus, dividend = 10 apples, divisor = 2 apples, quotient = 5, remainder = 0.

Repeated subtraction is the way to visualise the division of a thing.

Writing division expressions

The above situation can be written as

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \text{Remainder; specifically, } \frac{10 \text{ apples}}{2 \text{ apples}}$$

$$= 5 \text{ (2 apples)} + 0 \text{ (apples)}$$

Why do we need division?

This is an interesting question. For instance, why do we mostly cut a birthday cake into equal pieces (though not all people in attendance will eat the same quantity of cake)? Or, why do ketchup bottles come in packaging of 200 g, 500 g, or 1 Kg size (though no family or individual may be consuming exactly any of those quantities in one purchase cycle)?

Dividing things equally has many advantages:

- Easier and quicker to plan and regroup things (e.g., it would take no time to know the number of 5 ml bottles that can be made out of 234 litres of bulk nail polish, as compared to making a mix of 5 ml, 10 ml, 20 ml, 50 ml bottles),
- Making better choices in deciding actual consumption (one can take/buy exact multiples of the dividing group size that suits one's need/want),
- Fair sense of distribution.

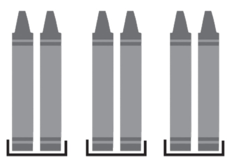
Division is not just repeated subtraction

The outcome of the subtraction operation is 'difference', unlike the outcomes of division – 'how many' (quotient) and 'what's left' (remainder); difference between quantities is no concern of division.

In fact, division is essentially used to express the quotient! Yes, repeated subtraction is arithmetically expressed as division, but we are concerned with how many repeated subtractions and not the difference.

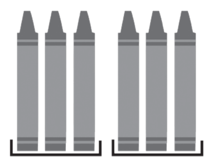
What happens when we divide a thing?

Each group created out of dividing a given thing become the new '1 unit'.



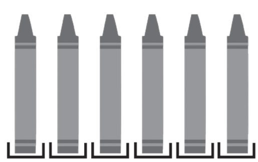
$$\frac{6 \text{ crayons}}{2 \text{ crayons}}$$

3 set of 2 crayons;
2 crayons is '1 unit'



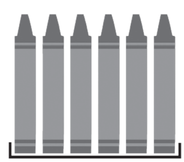
$$\frac{6 \text{ crayons}}{3 \text{ crayons}}$$

2 sets of 3 crayons;
3 crayons is '1 unit'



$$\frac{6 \text{ crayons}}{1 \text{ crayon}}$$

6 sets of 1 crayon;
1 crayon is '1 unit'



$$\frac{6 \text{ crayons}}{6 \text{ crayons}}$$

1 set of 6 crayons;
6 crayons is '1 unit'

Thus, the count of things changes due to the divisor becoming a unit of counting, but the quantity of things remains the same. For example,

$$\frac{8 \text{ apples}}{\frac{1}{2} \text{ apples}} \text{ is 16 half apples}$$

The total 8 apples wholes remains same.

Exploring dividend

Dividend is the quantity that is being divided/regrouped. It could be a collection of single things or packets of those things. For example, in triplets dozen, scores etc.

Exploring Divisor

Divisor is the size (quantity) of the group that the dividend will be divided into. Importantly, it has the same unit as the dividend, as illustrated:

$\frac{\text{8 pairs of apples}}{\text{2 pairs of apples}}$ ✓ $\quad$ $\frac{\text{8 pairs of apples}}{\text{2 apples}}$ ✗ $\quad$ $\frac{\text{9 dozen bananas}}{\text{3 dozen bananas}}$ ✓

Exploring quotient

It is the number of groups formed out of the dividend, each one the size of the divisor. It is written as a numeral, but it's like a multiplier (with the divisor being like the multiplicand), and indeed, for all division expressions, there is an equivalent multiplication expression:

$$\textbf{Quotient} \times \textbf{Divisor} + \textbf{Remainder} = \textbf{Dividend}$$

6 × 5 apples + 2 apples = 32 apples is same as the division expression

$\frac{\text{32 apples}}{\text{5 apples}} = 6\text{ (5 apples)} + 2\text{ apples}$

We know,

$$\frac{\textbf{Product}}{\textbf{Multiplicand}} = \textbf{Multiplier}$$

$\frac{\text{440 apples}}{\text{10 apples}}$ = 44 sets of 10 apples is the same as the multiplication expression 44 × 10 apples = 440 apples,

i.e, **multiplier × multiplicand = product**

Here, divisor is the multiplicand, quotient is the multiplier.

Exploring remainder

Remainder is the part of the dividend that can not be grouped; $\frac{35 \text{ apples}}{6 \text{ apples}}$ will leave 5 apples as remainder, and $\frac{36}{6}$ will leave no remainder.

The 'big' information in division expressions

The (quantity in) divisor determines the unit of outcome of any division operation.

Example 1

Let's evaluate 6 ÷ 1 ÷ 3 ÷ 2 vs 6 ÷ 6 (both give '1' as a quotient). Starting with

$$6 \div 1 \div 3 \div 2 = \cfrac{\cfrac{\cfrac{6}{1}}{3}}{2}$$

Because division is a binary operation, the steps to solving the questions are:

First step = $\frac{6}{1}$, the second step is $\cfrac{\cfrac{6}{1}}{3}$, and third step is $\cfrac{\cfrac{\cfrac{6}{1}}{3}}{2}$.

If 1 = 1 ball = ⚽

Then 6 = 6 balls = ⚽ ⚽ ⚽ ⚽ ⚽ ⚽

Step 1

$\frac{6}{1}$ = $\frac{⚽⚽⚽⚽⚽⚽}{⚽}$ = ⚽⚽⚽⚽⚽⚽ = 6 ⚽

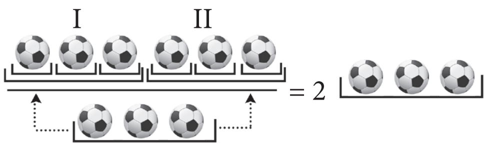
The unit of the quotient is ⚽, and ⚽ is the unit of the new dividend (as $\frac{6}{1}$ become the dividend).

Step 2

We know that the unit of the divisor is same as the unit of the dividend (). Thus, divisor as 3 would mean  as the divisor quantity.

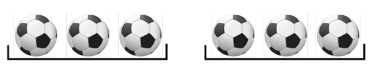
Thus,

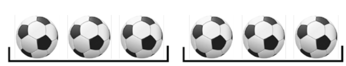
$$\frac{\frac{6}{1}}{3} =$$



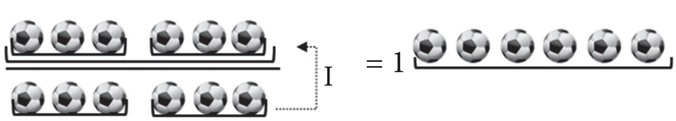
The unit of the quotient is , and it's the unit of the new divisor (as $\frac{\frac{6}{1}}{3}$ becomes the dividend).

Step 3

In $\frac{\frac{\frac{6}{1}}{3}}{2}$, we know $\frac{\frac{6}{1}}{3}$ = 

Divisor as 2 would mean  as the divisor quantity.

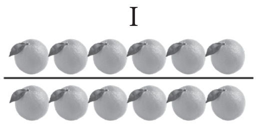
$$\frac{\frac{\frac{6}{1}}{3}}{2} =$$



= 1 set of 6 balls

Example 2

Let us evaluate $\frac{6}{6} = \frac{6 \text{ oranges}}{6 \text{ oranges}}$.

$\frac{6}{6} =$ 

$= 1$ 

= 1 set of 6 oranges

The magnitude of the quotient is 1.

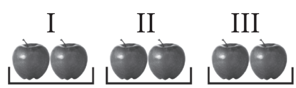
The unit of the quotient is  ('one set of 6 oranges').

However, $\frac{6 \text{ oranges}}{6 \text{ oranges}}$ is NOT 1, it is 1 set of 6 oranges.

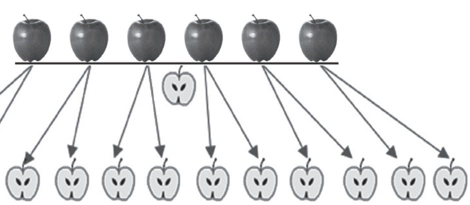
Example 3

Let us evaluate $6 \div 2$ and $6 \div \frac{1}{2}$.

Starting with, $\frac{6}{2} = \frac{6 \text{ apples}}{2 \text{ apples}}$

$\frac{6}{2} =$  $=$ 

$= 3$ 

$6 \div \frac{1}{2} = \frac{6}{\frac{1}{2}} = \frac{6 \text{ apples}}{\frac{1}{2} \text{ apple}} =$ 

$=$ 

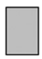
$= 12$ 

Example 4

Let us evaluate $8 \div \frac{1}{2} \div \frac{1}{5} \div 10$ or $\frac{\frac{\frac{8}{\frac{1}{2}}}{\frac{1}{5}}}{10}$.

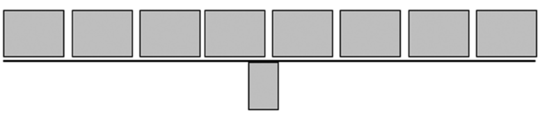
Step 1

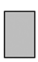
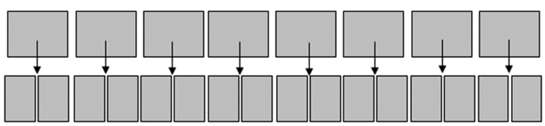
Let us visualise using a paper strip

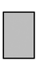
If '1' is , then $\frac{1}{2}$ is 

Then, 8 is 

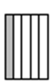
And $\frac{8}{\frac{1}{2}}$ is



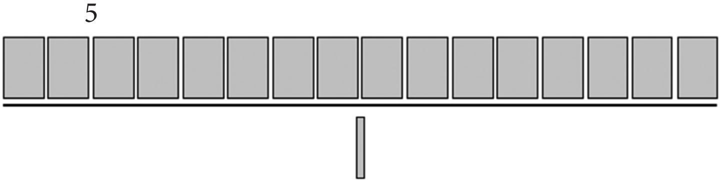
How many  in 

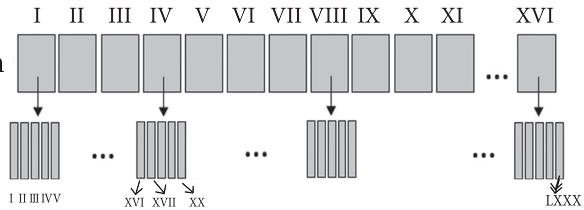
= 16 

Step 2

Now 1 = , so $\frac{1}{5}$ = 

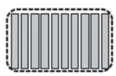
Then $\frac{16}{\frac{1}{5}}$ is

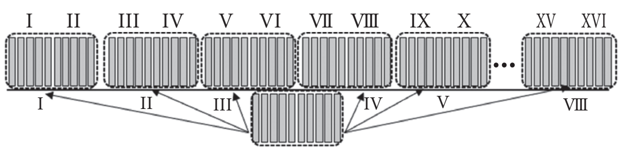


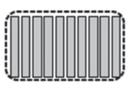
How many  in 

= 80 

Step 3

Now 1 =  and 10 = 

Next, $\frac{80}{10}$ = 

= 8 

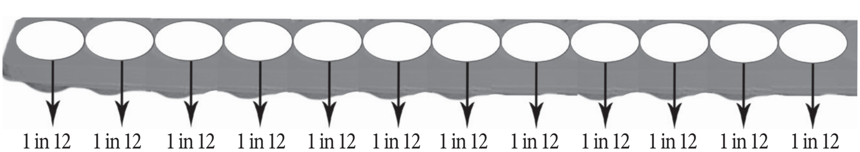
When quotient is 'less than 1'

Example 1

Let's consider $9 \div 12 = 9 \text{ oranges} \div 12 \text{ oranges} = \frac{9 \text{ oranges}}{12 \text{ oranges}}$.

$\frac{9}{12} = \frac{9 \text{ oranges}}{12 \text{ oranges}} =$ 

We can imagine the divisor as a packaging of 12 oranges:



The figure shows 12 empty slots in the packaging to keep 12 oranges.

Thus, $\frac{9 \text{ oranges}}{12 \text{ oranges}}$ is 

$$9 \text{ of } 1 \text{ in } 12 = 9 \times \frac{1}{12} = \frac{9}{12} \text{ of a box of 12 oranges}$$

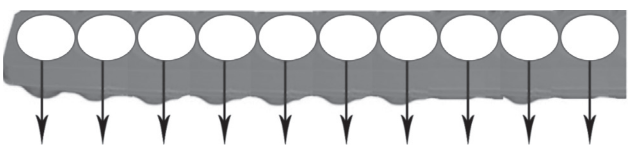
Here, quotient = 0 (12 oranges) and remainder = 9 oranges.

Example 2

Let's consider 5 ÷ 10 = 5 oranges ÷ 10 oranges = $\frac{5 \text{ oranges}}{10 \text{ oranges}}$.

$\frac{5}{10} = \frac{5 \text{ oranges}}{10 \text{ oranges}} =$ 

We can imagine the divisor as packaging of 10 oranges:



The picture shows 10 empty slots in the packaging to keep 10 oranges.

Thus, $\frac{5 \text{ oranges}}{10 \text{ oranges}}$ 

$$5 \text{ of } 1 \text{ in } 10 = 5 \times \frac{1}{10} = \frac{5}{10} \text{ of a box of 10 oranges}$$

Here, quotient = 0 (10 oranges) and remainder = 5 oranges.

Visualising $\frac{7}{3}$ as a division and as a fraction

Fraction and division represent very different physical reality.

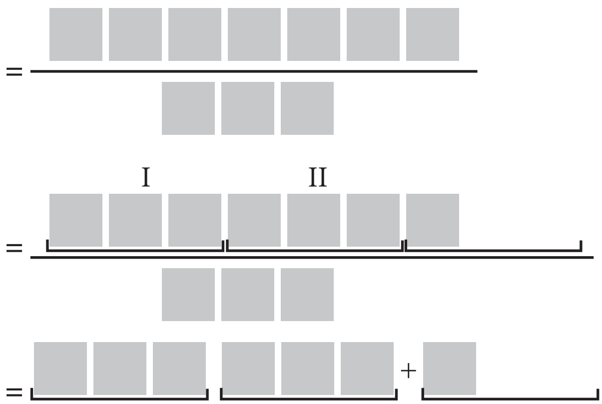
Let us visualise $\frac{7}{3}$ as a division and $\frac{7}{3}$ as a fraction.

Let 1 = ,

then 7 = , and 3 =

Evaluate $\frac{7}{3}$ as division.

$\frac{7}{3}$ as a division can be visualised as



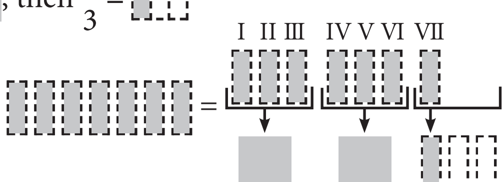
$\frac{7}{3}$ as a division means 2  as quotient and 1  as the remainder.

There are 7 in $\frac{7}{3}$ as a division.

Evaluate $\frac{7}{3}$ as fraction.

$\frac{7}{3}$ as a fraction is 7 of $\frac{1}{3} = 7 \times \frac{1}{3}$, it can be visualised as:

1 = , then $\frac{1}{3}$ = 

7 of $\frac{1}{3}$ = 

$\frac{7}{3}$ as a fraction means 2 + 

There are only 2 and 1 in $\frac{7}{3}$ as a fraction. It is a much smaller quantity compared to $\frac{7}{3}$ as a division.

The natural process of division

As we have already seen, division is about knowing or finding out the quotient – the number of all the possible groups that can be made out of the dividend quantity that are of the size of the divisor, recording any quantity left ungrouped as the remainder, and figuring all this out as quickly as possible!

To put it another way, the best way to find the number of groups is to keep counting and taking out quantities, same as the divisor, from the dividend till no more of such quantities can be taken out. Mathematically, it is to keep subtracting divisor from the dividend till it is feasible to do so (i.e., until the quantity left in the dividend is smaller than the quantity in the divisor); the number of feasible subtractions is the quotient.

Making subtractions faster

Which utensil may be the best for taking out rice from the container as quickly as possible?

Rice container   Utensils for taking out rice

The following two lessons are obvious from the question:

- We must takeaway, i.e., subtract, the largest quantity possible from the container (dividend) in each takeaway for quickly emptying the rice container. Mathematically, we must takeaway the largest multiple of the divisor.

- We must start from the biggest quantity of takeaway utensils for quickly emptying the rice container.

Subtracting numbers

Based on the lessons above, subtraction from a numeral is faster when:

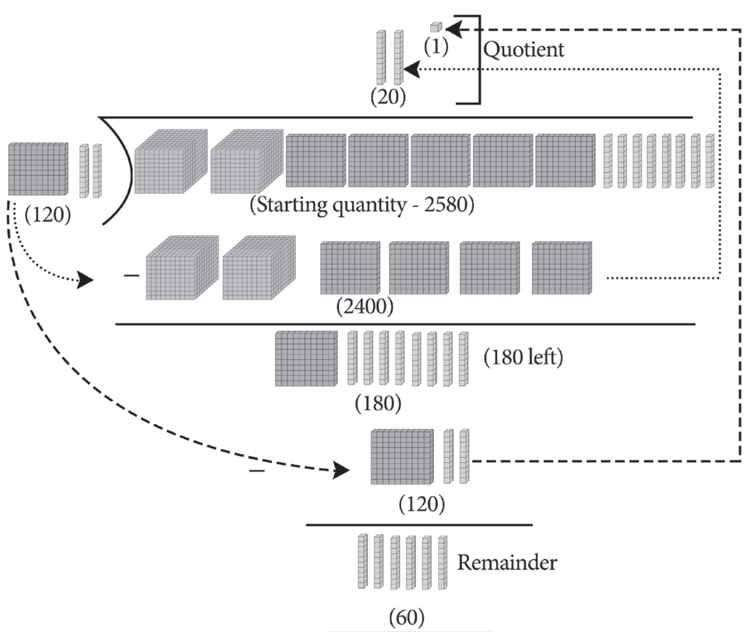
- The unit of subtraction is the divisor quantity.
- The multiple of the unit of subtraction are 'x 10', 'x 100', 'x 1000', etc. as that's the only possible quantities in decimal number system.
- The quantity of takeaway must be a multiple of the chosen utensil (that's the definition of division – quotient is multiple of divisors).

The long division method

Example

How much is 2580 ÷120?

$120\overline{)2580}$



Thus, quotient = 21
Remainder = 60

Dividing decimal numbers

Hopefully, it's easily appreciable that we can follow the same starting process as in multiplication and convert the given decimal numbers into whole numbers by multiplying it with 10 (or a multiple of 10). To keep the division expression equal, the dividend and the divisor must be multiplied by the same multiplier.

Example

How much is $\frac{45}{1.5}$?

Step 1

As the divisor is not a whole number, we multiply it with 10 to make it a whole number. The dividend too is multiplied by the same multiplier – 10 – to keep the division expression unchanged.

$$\frac{45}{1.5} = \frac{45 \times 10}{1.5 \times 10} = \frac{450}{15}$$

Step 2

Now we proceed as in the case of division of two whole numbers.

$$\frac{450}{15} = 30$$

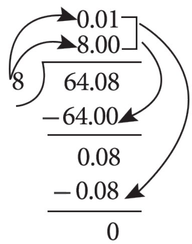
Note that we don't 'correct' the quotient which we had to do for the product in multiplication as the divisor and the dividend were multiplied by the same number.

Dividing a decimal number by a whole number

When dividing a decimal by a whole number, we divide as usual and place the decimal point in the quotient as it appears in the dividend.

Example

How much is $\frac{64.08}{8}$?



The quotient of division is the sum of the two individual quotients: **8.00 + 0.01 = 8.01.**

Dividing a decimal number by another decimal number

Here again, we multiply the dividend and the divisor with 10, or its multiple, to get whole numbers. To keep the division expression unchanged, the dividend and divisor are multiplied by the same multiplier. We then proceed in the same way as in the division of whole numbers to get the quotient and the remainder.

Example

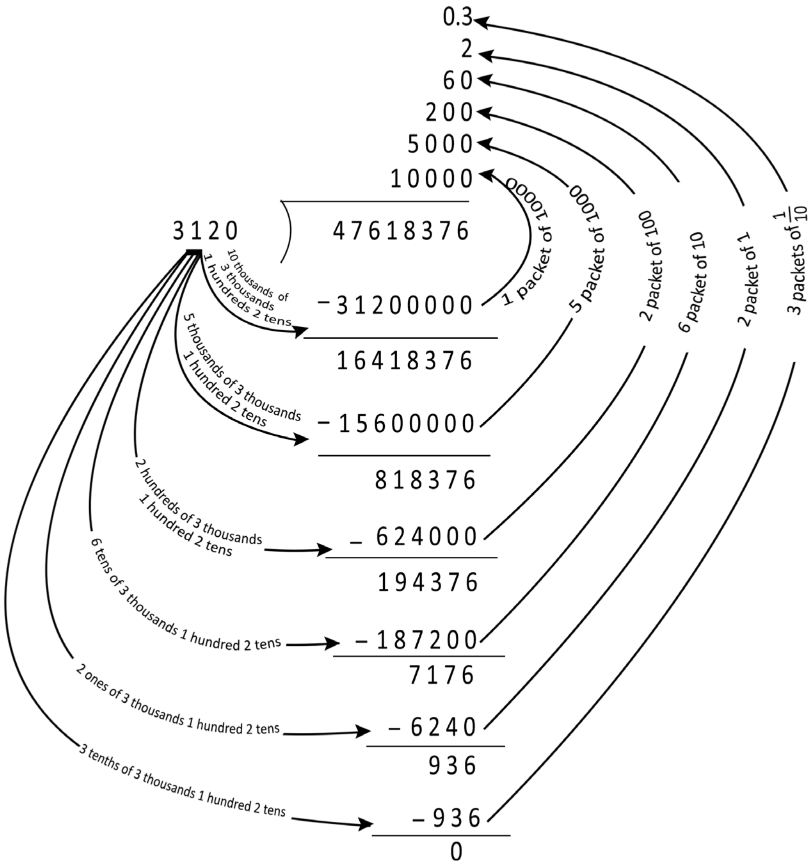
How much is $\frac{476183.76}{31.2}$?

Step 1

$$\frac{476183.76}{31.2} \times \frac{100}{100} \times \frac{10}{10} = \frac{47618376}{3120}$$

Step 2

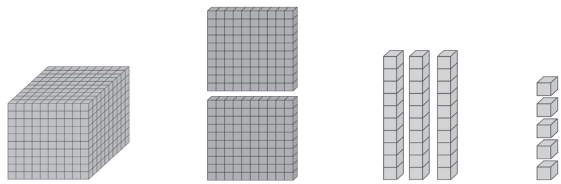
The long division method



Therefore, $\dfrac{476183.76}{31.2} = \dfrac{47618376}{3120} = 15262.3$

Why do we divide from the left?

The largest quantities are on the left side of numbers, because of this we can remove largest multiple of divisor quantity (the unit of packaging) at one go, making the process of division taster.
1 in 1235, the largest number block is on the left.



1 thousands 2 hundreds 3 tens 5 units

Division and other operations

Example

Evaluate $3 \times 4 \div 2 \div 3 \times 3 - 6$.

We'll evaluate the expression in an abstract arithmetical way and then the 'real', mathematical way (the correct way).

Let's examine the expression $3 \times 4 \div 2 \div 3 \times 3 - 6$ arithmetically.

Step 1

$3 \times 4 = 12$

Step 2

$$\frac{3 \times 4}{2} = \frac{12}{2} = 6$$

Step 3

$$\frac{\frac{3 \times 4}{2}}{3} = \frac{6}{3} = 2$$

Step 4

$$\frac{\frac{3 \times 4}{2}}{3} \times 3 = 2 \times 3 = 6$$

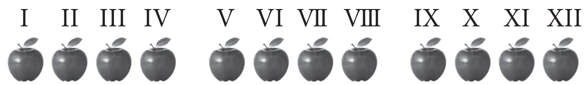
Step 5

$$\frac{\frac{3 \times 4}{2}}{3} \times 3 - 6 = 6 - 6 = 0$$

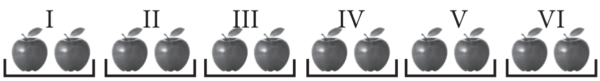
Let's examine the expression $3 \times 4 \div 2 \div 3 \times 3 - 6$ mathematically.

In the mathematical way of expression, it's important to understand that the part of an expression before the minus symbol is the minuend and the part after the minus symbol is the subtrahend. Obtaining the minuend:

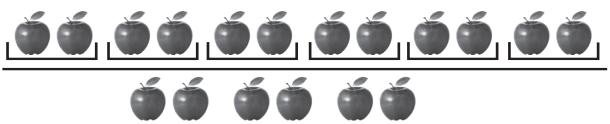
Step 1

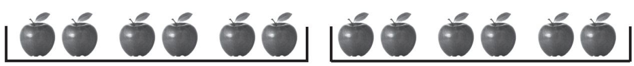
$3 \times 4 =$ 

Step 2

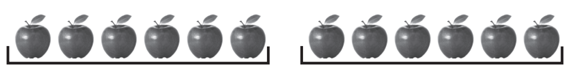
$\dfrac{3 \times 4}{2} =$ 

Step 3

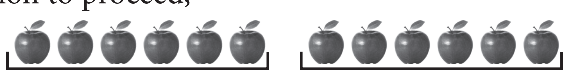
$\dfrac{\frac{3 \times 4}{2}}{3} =$ 

$=$ 

Step 4

$\dfrac{\frac{3 \times 4}{2}}{3} \times 3 =$  ...6

As the minuend and the subtrahend has to be the same unit for subtraction operation to proceed,

the subtrahend 6 =  ...6

Step 5

$\dfrac{\frac{3 \times 4}{2}}{3} \times 3 - 6 = 6$  $- 6$ 

$= 0$

Did you enjoy reading about division? Explore more on division with 'Foundations of Division (Mathematics as a Language)' available on Amazon.