## Септ Pęcent mitiennitics <br> Ace 'Al \& Industry 4.0 Era' Amplify Your AIQ \& Wealth

Uses AERO, or, US 'Common Core Plus' Mathematics Curriculum, for the
 overall academic planning.
The choice of AERO is to ensure progressive integrity of MATHEMATICAL THINKING in the lessons.

However, the lesson content shall be based the unique ' $\mathrm{A}-\mathrm{Z}$ ' Mathematics that we have authored.
'A - Z ' Mathematics is THE FIRST-EVER mathematics learning resource that is threaded by MATHEMATICAL LOGIC.
The very conception and creation of 'A - Z' Mathematics implied truly $100 \%$ Mathematics, that mathematises thinking.

The combining of the curricular detailing of mathematics by AERO, and the visualisation and verbalisation of 'A - Z' Mathematics, is the magic, awaited for 200 years.

CURRICULUM FRAMEWORK FOR MATH

## About AERO

American Education Reaches Out (AERO) is a project supported by the United States Department of Education's Office of Overseas Schools, which establishes an implementation framework for international American schools which offer a standards-based U.S. curriculum.

Aligned with Common Core standards and Next Generation Science Standards, AERO is considered to be "Common Core Plus," providing an "enduring understanding, essential questions and learning progression."

## Why US Common Core Mathematics?

It is one of the best curriculum for learning MATHEMATICAL THINKING. That is, to MATHEMATISE THINKING.
In these times of ever-sharpening Artificial (General) Intelligence, mathematised thinking is the only way to be successful. The typical rote, methodised, logic-less mathematics is of no value.

This curriculum will also empower foundation for success in ALL School System Curricula, globally.

## The Background

One of the primary reasons, for poor mathematics education, is the lack of distinction, between mathematics content (largely methods, and practice), and mathematical thinking. This ambiguity, is also reflected, in assessment, and in evaluation.

The education system assumes, that teaching mathematics compulsorily, is enough, by itself, to develop mathematical thinking.

But, why content, and thinking, are different? That is, why the methods, and ceaseless practice, in mathematics education, does not promote mathematised mind.

For, the content does not focus on logic, process, reasoning, and the history of relevant mathematical concepts.
Briefly, mathematical thinking is not, thinking about the subject matter of mathematics, but, a way of looking, at situations, and conditions. It is critical for success in all academic 'subjects.'

## AERO Mathematics Standards

## Progressions 3-5

| Domain: Number and Operations in Base Ten | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Use place value understanding and properties of operations to perform multi-digit arithmetic | AERO. 3.NBT. 1 DOK 1 <br> Use place value understanding to round whole numbers to the nearest 10 or 100. | AERO. 4.NBT. 3 DOK 1 <br> Use place value understanding to round multi-digit whole numbers to any place | AERO. 5.NBT. $4 \quad$ DOK 1 Use place value understanding to round decimals to any place. |
|  | AERO. 3.NBT. 2 DOK 1,2 <br> Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. | AERO. 4.NBT. $4 \quad$ DOK 1 Fluently add and subtract multi-digit whole numbers using the standard algorithm. | AERO. 5.NBT. 5 <br> DOK 1 <br> Fluently multiply multi-digit whole numbers using the standard algorithm. |
|  | $\begin{aligned} & \text { AERO. 3.NBT. } 3 \quad \text { DOK 1,2 } \\ & \text { Multiply one-digit whole numbers by } \\ & \text { multiples of } 10 \text { in the range } 10-90 \\ & (e . g ., 9 \times 80,5 \times 60) \text { using strategies } \\ & \text { based on place value and properties } \\ & \text { of operations. } \end{aligned}$ | AERO. 4.NBT. 5 <br> Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. | AERO. 5.NBT. 2 DOK 1,2 <br> Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10 . Use whole-number exponents to denote powers of 10 . |
|  | AERO. 3.OA.2 DOK 1,2 <br> Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$. | AERO. 4.NBT. 6 <br> Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. | AERO. 5.NBT.6 DOK 1,2 <br> Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |


| Domain: Number and Operations in Base Ten | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Use place value understanding and properties of operations to perform multi-digit arithmetic | AERO. 4.NF.5 DOK 1  <br>  Express a fraction with denominator <br> 10 as an equivalent fraction with <br> denominator 100, and use this <br> technique to add two fractions with <br> respective denominators 10 and 100. <br> For example, express $3 / 10$ as $30 / 100$, <br> and add $3 / 10+4 / 100=34 / 100$. |  | AERO. 5.NBT. 7 DOK 1,2,3 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. |
|  |  | AERO. 4.NF. 6 DOK 1 Use decimal notation for fractions with denominators 10 or 100 . For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. |  |
|  |  | AERO. 4.NBT. 1 <br> Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70=10$ by applying concepts of place value and division. | AERO. 5.NBT. 1 DOK 1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1 / 10$ of what it represents in the place to its left. |
|  |  |  | AERO. 5.NBT. 3 DOK 1 <br> Read, write, and compare decimals to thousandths. |
|  |  |  | $\begin{aligned} & \text { AERO. 5.NBT.3a } \\ & \text { Read and write decimals to } \\ & \text { thousandths using base-ten numerals, } \\ & \text { number names, and expanded form, } \\ & \text { e. } 9 ., 347.392=3 \times 100+4 \times 10+7 \times \\ & 1+3 \times(1 / 10)+9 \times(1 / 100)+2 \times \\ & (1 / 1000) . \end{aligned}$ |
|  |  |  | AERO. 5.NBT.3a <br> Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, $\text { e.g., } 347.392=3 \times 100+4 \times 10+7 \times 2$ $1+3 \times(1 / 10)+9 \times(1 / 100)+2 \times$ <br> (1/1000). |


| Domain: Number and Operations in Base Ten | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Generalize place value understanding for multi-digit whole numbers and decimals to hundredths |  | AERO. 4.NF. 7 DOK 1,2,3 <br> Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. | AERO. 5.NBT.3b DOK 1 <br> Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons |
| Represent and solve problems involving multiplication and division. | AERO. 3.OA. 4 DOK 1,2 <br> Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ?=48,5=-\div 3,6 \times 6=$ ? |  | AERO. 5.OA. 2 <br> Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7 , then multiply by 2 " as $2 \times(8+7)$. <br> Recognize that $3 \times(18932+921)$ is three times as large as $18932+921$, without having to calculate the indicated sum or product. |
|  | AERO. 3.OA. 6 DOK 1,2 <br> Understand division as an unknownfactor problem. For example, find 32 8 by finding the number that makes 32 when multiplied by 8 . |  |  |
|  | AERO. 3.OA. 3 DOK 1,2 <br> Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem |  |  |



| Domain: Operations and Algebraic Thinking | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Understand properties of multiplication and the relationship between multiplication and division. | AERO. 3.OA. 5 <br> Apply properties of operations as strategies to multiply and divide. <br> Examples: If $6 \times 4=24$ is known, then $4 \times 6=24$ is also known. <br> (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5=15$, then $15 \times 2=30$, or by $5 \times 2=10$, then $3 \times 10=30$. <br> (Associative property of multiplication.) Knowing that $8 \times 5=$ 40 and $8 \times 2=16$, one can find $8 \times 7$ as $8 \times(5+2)=(8 \times 5)+(8 \times 2)=40$ $+16=56$. (Distributive property.) |  | AERO. 5.OA. 1 DOK 1 <br> Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols |
|  | AERO. 3.OA.1 DOK 1,2 <br> Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as $5 \times 7$ | AERO. 4.OA.1 DOK 1,2 Interpret a multiplication equation as a comparison, e.g., interpret $35=5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5 . Represent verbal statements of multiplicative comparisons as multiplication equations |  |
| Multiply and divide within 100. | AERO. 3.OA. 7 <br> Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5$ $=40$, one knows $40 \div 5=8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. |  |  |


| Domain: Operations and <br> Algebraic Thinking | $\mathbf{c}$ |  |  |
| :--- | :--- | :--- | :--- |
| Solve problems involving <br> the four operations, and <br> identify and explain patterns <br> in arithmetic | AERO. 3.OA.8 DOK 1,2,3 <br> Solve two-step word problems using <br> the four operations. Represent these <br> problems using equations with a letter <br> standing for the unknown quantity. <br> Assess the reasonableness of <br> answers using mental computation <br> and estimation strategies including <br> rounding | AERO. 4.OA.3 <br> Solve multistep word problems posed <br> with whole numbers and having <br> whole-number answers using the four <br> operations, including problems in <br> which remainders must be interpreted. <br> Represent these problems using <br> equations with a letter standing for the <br> unknown quantity. Assess the <br> reasonableness of answers using <br> mental computation and estimation <br> strategies including rounding. | $\mathbf{5}$ |


| Domain: Operations and Algebraic Thinking | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Gain familiarity with factors and multiples. |  | AERO. 4.OA. 4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1 100 is a multiple of a given one-digit number. Determine whether a given whole number in the range $1-100$ is prime or composite. |  |
| Generate and analyze patterns. |  | AERO. 4.OA. 5 DOK 1,2 <br> Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1 , generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. | AERO. 5.OA. 3 <br> Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0 , and given the rule "Add 6" and the starting number 0 , generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. |



| Domain: Numbers and Operations- Fractions | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Develop understanding of fractions as numbers. | AERO. 3.NF.3a DOK 1,2,3 Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line |  |  |
|  | AERO. 3.NF.3b DOK 1,2.3 <br> Recognize and generate simple equivalent fractions, e.g., $1 / 2=2 / 4$, $4 / 6=2 / 3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. |  |  |
|  | AERO. 3.NF.3c DOK 1,2,3 <br> Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3=$ $3 / 1$; recognize that $6 / 1=6$; locate $4 / 4$ and 1 at the same point of a number line diagram. |  |  |
|  | AERO. 3.NF.3d DOK 1,2.3 <br> Compare two fractions with the same numerator or the same denominator by reasoning about their size. <br> Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, $=$, or <, and justify the conclusions, e.g., by using a visual fraction model. | AERO. 4.NF. 2 <br> Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1 / 2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. |  |


| Domain: Numbers and Operations- Fractions | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Build fractions from unit fractions. |  | AERO. 4.NF. 3 DOK 1,2,3 Understand a fraction $\mathrm{a} / \mathrm{b}$ with $\mathrm{a}>1$ as a sum of fractions $1 / \mathrm{b}$. |  |
| Use equivalent fractions as a strategy to add and subtract fractions. |  | AERO. 4.NF.3a DOK 1,2,3 <br> Understand addition and subtraction of fractions as joining and separating parts referring to the same whole | AERO. 5.NF. 1 DOK 1 <br> Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2 / 3+5 / 4$ $=8 / 12+15 / 12=23 / 12$. (In general, $a / b+c / d=(a d+b c) / b d$. |
|  |  | AERO. 4.NF.3b DOK 1,2,3 <br> Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3 / 8=$ $1 / 8+1 / 8+1 / 8 ; 3 / 8=1 / 8+2 / 8 ; 21 / 8$ $=1+1+1 / 8=8 / 8+8 / 8+1 / 8$. |  |
|  |  | AERO. 4.NF.3c DOK $1,2,3$ <br> Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. |  |



| Domain: Numbers and Operations- Fractions | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Apply and extend previous understandings of multiplication and division. |  | AERO. 4.NF.4c <br> Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3 / 8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? | AERO. 5.NF. 3 <br> Interpret a fraction as division of the numerator by the denominator $(\mathrm{a} / \mathrm{b}=\mathrm{a}$ <br> $\div$ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3 / 4$ as the result of dividing 3 by 4 , noting that $3 / 4$ multiplied by 4 equals 3 , and that when 3 wholes are shared equally among 4 people each person has a share of size $3 / 4$. If 9 people want to share a 50 -pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? |
|  |  |  | AERO. 5.F.4b DOK 1,2 <br> Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas |
|  |  |  | AERO. 5.F.5a DOK 1,2,3 Interpret multiplication as scaling (resizing), by Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. |


| Domain: Numbers and Operations- Fractions | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Apply and extend previous understandings of multiplication and division. |  |  | AERO. 5.NF.5b <br> Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a / b=$ $(n \times a) /(n \times b)$ to the effect of multiplying $\mathrm{a} / \mathrm{b}$ by 1 . |
|  |  |  | AERO. 5.NF. 6 DOK 1,2 <br> Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. |
|  |  |  | AERO. 5.NF. 7 DOK 1,2 <br> Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions |
|  |  |  | AERO. 5.NF.7a DOK 1,2 Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1 / 3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1 / 3) \div 4=1 / 12$ because $(1 / 12) \times$ $4=1 / 3$. |


| Domain: Numbers and Operations- Fractions | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Apply and extend previous understandings of multiplication and division |  |  | AERO. 5.NF.7b <br> Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div(1 / 5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div(1 / 5)=20$ because $20 \times(1 / 5)$ $=4$. |
|  |  |  | AERO. 5.NF.7c DOK 1,2 <br> Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1 / 2$ lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins? |


| Domain: Measurement and Data | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Solve problems involving measurement and estimation. | AERO. 3.MD. 1 DOK 1,2 <br> Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram |  |  |
|  | AERO. 3.MD. 2 <br> DOK 1,2 <br> Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (I). 1 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. |  |  |
| Solve problems involving measurement and conversion of measurements. |  | AERO. 4.MD. 1 <br> Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml ; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a twocolumn table. For example, know that 1 ft is 12 times as long as 1 in . Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), | AERO. 5.MD. 1 DOK 1,2 <br> Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m ), and use these conversions in solving multistep, real world problems. |


| Domain: Measurement and Data | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Solve problems involving measurement and conversion of measurements |  | AERO. 4.MD. 2 <br> Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. |  |
|  |  | AERO. 4.MD. 3 DOK 1,2 <br> Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. |  |
| Represent and interpret data. | AERO. 3.MD. 3 $\qquad$ <br> Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. | AERO. 4.MD. 4 DOK 1,2 <br> Make a line plot to display a data set of measurements in fractions of a unit ( $1 / 2,1 / 4,1 / 8$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. | AERO. 5.NF. 2 <br> DOK 1,2,3 <br> Make a line plot to display a data set of measurements in fractions of a unit ( $1 / 2,1 / 4,1 / 8$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. |


| Domain: Measurement and Data | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Represent and interpret data | AERO. 3.MD. 4 DOK 2 <br> Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate unitswhole numbers, halves, or quarters. |  |  |
| Geometric measurement: understand concepts of area and relate area to multiplication and to addition. <br> Geometric measurement: understand concepts of volume. | AERO. 3.MD.5 DOK 1,2 <br> Recognize area as an attribute of plane figures and understand concepts of area measurement. |  | AERO. 5.MD. 3 DOK 1 <br> Recognize volume as an attribute of solid figures and understand concepts of volume measurement. |
|  | AERO. 3.MD.5a <br> DOK 1,2 <br> A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. |  | AERO. 5.MD.3a DOK 1 <br> A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. |
|  | AERO. 3.MD.5b DOK 1,2 <br> A plane figure which can be covered without gaps or overlaps by $n$ unit squares is said to have an area of $n$ square units. |  | AERO. 5.MD.3b DOK 1 <br> A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of $n$ cubic units. |
|  | AERO.3.MD. $6 \quad$ DOK 1,2 Measure areas by counting unit squares (square cm , square m, square in, square ft , and improvised units) |  | AERO.5.MD. 4 DOK 1,2 <br> Measure volumes by counting unit cubes, using cubic cm , cubic in, cubic ft , and improvised units. |


| Domain: Measurement and Data | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Geometric measurement: understand concepts of volume. | AERO. 3.MD. $7 \quad$ DOK 1,2 Relate area to the operations of multiplication and addition. |  | AERO. 5.MD. 5 DOK 1,2 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume |
|  | AERO. 3.MD.7a DOK 1,2 Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. |  | AERO. 5.MD.5a DOK 1,2 Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold wholenumber products as volumes, e.g., to represent the associative property of multiplication. |
|  | AERO. 3.MD.7b DOK 1,2 <br> Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning |  | AERO. 5.MD.5b DOK 1,2 Apply the formulas $\mathrm{V}=\mathrm{I} \times \mathrm{w} \times \mathrm{h}$ and V $=\mathrm{b} \times \mathrm{h}$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. |
|  | AERO. 3.MD.7c DOK 1,2 <br> Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b+$ c is the sum of $\mathrm{a} \times \mathrm{b}$ and $\mathrm{a} \times \mathrm{c}$. Use area models to represent the distributive property in mathematical reasoning. |  |  |


| Domain: Measurement and Data | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Geometric measurement: understand concepts of volume. | AERO. 3.MD.7d DOK 1,2Recognize area as additive. Findareas of rectilinear figures bydecomposing them into non-overlapping rectangles and adding theareas of the non-overlapping parts,applying this technique to solve realworld problems. |  | AERO. 5.MD.5c DOK 1,2 Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. |
|  |  |  | AERO. 5.MD. 3 DOK 1 <br> Recognize volume as an attribute of solid figures and understand concepts of volume measurement. |
| Geometric measurement: recognize perimeter. | AERO. 3.MD. 8 <br> Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. |  |  |
| Geometric measurement: understand concepts of angle and measure angles. |  | AERO. 4.MD. 5 DOK 1 <br> Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: |  |


| Domain: Measurement and Data | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Geometric measurement: understand concepts of angle and measure angles. |  | AERO. 4.MD.5a DOK 1 <br> An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1 / 360$ of a circle is called a "one-degree angle," and can be used to measure angles. |  |
|  |  | AERO. 4.MD.5b DOK 1 <br> An angle that turns through $n$ onedegree angles is said to have an angle measure of $n$ degrees. |  |
|  |  | AERO. 4.MD. 6 DOK 1 <br> Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. |  |
|  |  | AERO. 4.MD. 7 DOK 1,2 <br> Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. <br> Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. |  |


| Domain: Geometry | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Reason with shapes and their attributes. | AERO. 3.G. 1 DOK 1,2 <br> Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. |  | AERO. 5.G. 3 DOK 1,2 Classify two-dimensional figures into categories based on their properties. |
|  | AERO. 3.G. 2 DOK 1,2 $\square$ <br> Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $1 / 4$ of the area of the shape. |  |  |
|  |  | AERO. 4.G. 1 DOK 1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. |  |
|  |  | AERO. 4.G. 2 DOK 1,2 <br> Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. |  |
| AERO Mathematics Standards Ju |  | AERO. 4.G.2 DOK 1,2 <br> Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify | 20 |


| Domain: Geometry | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| Graph points on the coordinate plane to solve real-world and mathematical problems. |  | AERO. 4.G.3 DOK 1 <br> Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. |  |
|  |  |  | .AERO. 5.G. 1 DOK 1 <br> Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., $x$-axis and x -coordinate, y -axis and y coordinate). |
|  |  |  | AERO. 5.G.2 $\quad$ DOK 1,2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. |
|  |  |  | AERO. 5.G.4 DOK 1,2 Classify two-dimensional figures in a hierarchy based on properties |


| Mathematical Practices | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| 1. Make sense of problems and persevere in solving them. | Explain to themselves the meaning of a problem and look for ways to solve it. <br> May use concrete objects or pictures to help them conceptualize and solve problems. <br> May check their thinking by asking themselves, "Does this make sense?" <br> Listen to the strategies of others and will try different approaches. <br> Will use another method to check their answers. | Know that doing mathematics involves solving problems and discussing how they solved them. <br> Explain to themselves the meaning of a problem and look for ways to solve it. <br> May use concrete objects or pictures to help them conceptualize and solve problems. <br> May check their thinking by asking themselves, "Does this make sense?" <br> Listen to the strategies of others and will try different approaches. <br> Will use another method to check their answers. | Solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. <br> Solve problems related to volume and measurement conversions. <br> Seek the meaning of a problem and look for efficient ways to represent and solve it. <br> Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". |


| Mathematical Practices | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| 2. Reason abstractly and quantitatively. | Recognize that a number represents <br> a specific quantity. <br> Connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. | Recognize that a number represents <br> a specific quantity. <br> Connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. <br> Extend this understanding from whole numbers to their work with fractions and decimals. <br> Write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. | Recognize that a number represents <br> a specific quantity. <br> Connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. <br> Extend this understanding from whole numbers to their work with fractions and decimals. <br> Write simple expressions that record calculations with numbers and represent or round numbers using place value concepts. |


| Mathematical Practices | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| 3. Construct viable arguments and critique the reasoning of others. | May construct arguments using concrete referents, such as objects, pictures, and drawings. <br> Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" <br> Explain their thinking to others and respond to others' thinking. | May construct arguments using concrete referents, such as objects, pictures, and drawings. <br> Explain their thinking and make connections between models and equations. <br> Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" <br> Explain their thinking to others and respond to others' thinking. | Construct arguments using concrete referents, such as objects, pictures, and drawings. <br> Explain calculations based upon models and properties of operations and rules that generate patterns. <br> Demonstrate and explain the relationship between volume and multiplication. <br> Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" <br> Explain their thinking to others and respond to others' thinking. |


| Mathematical Practices | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| 4. Model with mathematics. | Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. <br> -Connect the different representations and explain the connections. <br> Evaluate their results in the context of the situation and reflect on whether the results make sense. | Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. <br> Connect the different representations and explain the connections. <br> Use all of these representations as needed. <br> Evaluate their results in the context of the situation and reflect on whether the results make sense. | Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. <br> Connect the different representations and explain the connections. <br> Use all of these representations as needed. <br> Evaluate their results in the context of the situation and whether the results make sense. <br> Evaluate the utility of models to determine which models are most useful and efficient to solve problems. |


| Mathematical Practices | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| 5. Use appropriate tools strategically. | Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For EXAMPLE, they may use graph paper to find all the possible rectangles that have a given perimeter. <br> Compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. | Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. <br> Use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. | Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. <br> Use graph paper to accurately create graphs and solve problems or make predictions from real world data. |
| 6. Attend to precision. | Use clear and precise language in their discussions with others and in their own reasoning. <br> Are careful about specifying units of measure and state the meaning of the symbols they choose. For example, when figuring out the area of a rectangle they record their answers in square units. | Develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. <br> Are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot. | Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. <br> Use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. <br> Are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units. |
|  |  |  |  |
|  |  |  |  |


| $\begin{array}{l}\text { Mathematical Practices }\end{array}$ | $\mathbf{3}$ | $\mathbf{4}$ |  |
| :--- | :--- | :--- | :--- |
| $\begin{array}{l}\text { 7. Look for and make use of } \\ \text { structure. (Deductive } \\ \text { Reasoning) }\end{array}$ | $\begin{array}{l}\text { Look closely to discover a pattern or } \\ \text { structure. For example, students use } \\ \text { properties of operations as } \\ \text { strategies to multiply and divide } \\ \text { (commutative and distributive } \\ \text { properties). }\end{array}$ | $\begin{array}{l}\text { Look closely to discover a pattern or } \\ \text { structure. For instance, students use } \\ \text { properties of operations to explain } \\ \text { calculations (partial products model). }\end{array}$ | $\begin{array}{l}\text { Look closely to discover a pattern or } \\ \text { structure. For instance, students use } \\ \text { properties of operations as } \\ \text { strategies to add, subtract, multiply } \\ \text { and divide with whole numbers, } \\ \text { fractions, and decimals. }\end{array}$ |
| Relate representations of counting |  |  |  |
| problems such as tree diagrams and |  |  |  |
| arrays to the multiplication principal |  |  |  |
| of counting. |  |  |  |\(\left.\} \begin{array}{l}Examine numerical patterns and <br>

relate them to a rule or a graphical <br>
representation.\end{array}\right\}\)

